14 transport

No assigned homework problems.
Suggested problems for further study:

14.1: Thermal conductivity of a dilute gas
Exercise 13.6 from Reif's book

14.2: Thermal conductivity of conduction electrons in a metal
Exercise 13.11 from Reif's book
You may want to read Sec. 12.4 from Reif's book where you can find an qualitative estimate of the coefficient of thermal conductivity.

14.3: Wiedemann-Franz law
Exercise 13.12 from Reif's book

14.4: Mixing of gases
Exercise 12.15 from Reif's book.
Hint: use the estimate $l \approx 3 \times 10^{-7}$m for the mean free path of a gas at room temperature and atmospheric pressure, see Reif, Eq. (12.2.14).

14.5: One-dimensional conductor
It is possible to fabricate samples that behave as defect-free one-dimensional conductors. Examples are "quantum wires" in semiconductor heterostructures, or carbon nanotubes. At very low temperatures, interactions lead to a strongly correlated electronic ground state (the so-called "Luttinger Liquid"). However, at not too low temperatures, a description in terms of the Boltzmann equation may still be appropriate.

In a one-dimensional conductor, the relevant momenta p are in the vicinity of either p_F or $-p_F$, see figure 1. (The 'Fermi momentum' is the momentum of electrons with energy equal to the Fermi energy.) One usually refers to the two types of excitations as "right movers" and "left movers". As a result, in one dimension the distribution function f can be represented by two functions f_R and f_L that each depend on the kinetic energy ε of the electrons only. In the absence of collisions, the Boltzmann equation then reads

$$\frac{\partial f_R}{\partial t} + v_F \frac{\partial f_R}{\partial x} + eEv_F \frac{\partial f_R}{\partial \varepsilon} \quad = \quad 0, \tag{1}$$

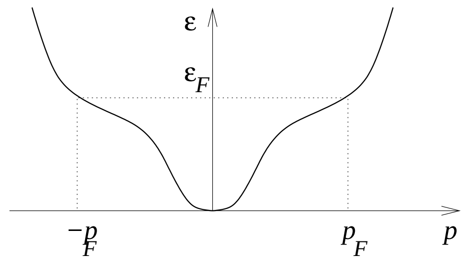

Figure 1: Dispersion relation of a one dimensional metal. All relevant momenta are close to p_F and $-p_F$.

$$\frac{\partial f_L}{\partial t} - v_F \frac{\partial f_L}{\partial x} - eEv_F \frac{\partial f_L}{\partial \varepsilon} = 0, \tag{2}$$

where v_F is the Fermi velocity. (The 'Fermi velocity' is the velocity of electrons moving at the Fermi energy.)

(a) Show that in one dimension, the Fermi velocity is related to the density of states at the Fermi level $\nu(\varepsilon_F)$.

Consider a one dimensional conductor in which the electric field exists only within a segment of length L. Assume that all electrons entering that segment are in thermal equilibrium, *i.e.,* their distribution function $f(\varepsilon)$ is the Fermi function. Also assume that the one dimensional conductor is perfect, *i.e.,* that there are no collisions between the electrons or between electrons and impurities or defects.

(b) Solve the Boltzmann equation to find the electrical current in the conductor.

(c) What does your answer to (b) imply if expressed in terms of a "conductivity"? Does your answer make sense?

(d) One can also express the answer to (b) in terms of the "conductance" G, defined as the quotient of the current I and the voltage $V = EL$ over the conductor. What conductance do you find? What physical properties does it depend on?