

# Cornell Center for Materials Research

<http://www.ccmr.cornell.edu/>

Activity: “How can X-rays be useful in studying crystals?”

Lab applications: Coherent light diffraction and interference

Author: Ulugbek T. Akhmedov, Physics teacher

School: Baruch College Campus High School, New York NY

CCMR RET  
Summer 2004

**CURRICULUM PROJECT: Light Diffraction and Interference**  
(Estimated total time double period – 2x50 min)

**SUMMARY:**

Regents Physics:

Wave Phenomena is one of the units of the Regents Physics curriculum. Regents Physics is mandatory for all 11th grade students at Baruch College Campus High School (BCCHS), while many schools offer the course as an elective for 12<sup>th</sup> graders.

**STANDARDS:**

All of the following New York State Learning Standards in Mathematics, Science, and Technology will apply:

**Standard 1: Analysis, Inquiry, and Design**

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, to seek answers, and develop solutions.

**Standard 2: Information Systems**

Students will access, generate, process, and transfer information using appropriate technologies.

**Standard 3: Mathematics**

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

**Standard 4: Science**

Students will understand and apply scientific concepts, principles, and theories pertaining to the physical setting and living environment and recognize the historical development of ideas in science.

**Standard 5: Technology**

Students will apply technological knowledge and skills to design, construct, use, and evaluate products and systems to satisfy human and environmental needs.

**Standard 6: Interconnectedness: Common Themes**

Students will understand the relationships and common themes that connect mathematics, science, and technology and apply the themes to these and other areas of learning.

## **Standard 7: Interdisciplinary Problem Solving**

Students will apply the knowledge and thinking skills of mathematics, science, and technology to address real-life problems and make informed decisions.

### PRIOR KNOWLEDGE:

Students can describe what a wave is, as well as its features like wavelength, amplitude, frequency, and period. Students are expected to know and recognize phase in relation to the mathematics-B topic about shifting graphs along the x-axis.

Additionally, students are expected to have solid understanding of wave superimposing: constructive and destructive superimpositions.

### WAVE PHENOMENA:

- Wavelength, amplitude, frequency, and period
- Reflection
- Refraction
- Single-slit and double-slit diffraction
- Multiple slit diffraction
- Constructive and destructive interference

### OBJECTIVES FOR TEACHERS:

Students are expected to understand basic wave phenomena using simple wave models. Although diffraction grating formula and Bragg Equation are not required, application of these will help teachers:

- a) engage students in making connections between seemingly abstract and/or “useless” phenomena and Material Science applications;
- b) assess students conceptual understanding as the students will have to explain how their knowledge about the wave phenomena becomes useful;
- c) expand the horizons of students’ knowledge and engage them into further study of concepts and application around the wave phenomena discussed in class

### **SAFETY PRECAUTIONS**

As the activities involve laser beams, it is strongly recommended to inform students about health hazards of amplified radiation especially when eyes are exposed to it. Even seemingly low power lasers can cause temporary or even permanent damage to eye retina because the beam is a very concentrated energy. The actual setup may be assembled by the students and experiments with various diffraction gratings can be conducted under the teacher’s supervision. However, it is strongly recommended that operation of the laser source should be solely controlled by the teacher. The teacher should be able to turn the laser on and off at any time and should not rely on students doing this. The measurements can be done with the help of the digital camera as detailed further. Safety training can be role played prior to this lesson by the students acting as real scientists planning to use safeguards to protect themselves and the environment from hazards.

### OBJECTIVES FOR STUDENTS:

1. After learning about diffraction phenomenon, students will use their prior knowledge and should be able to apply the concepts to identify a) wavelength of the laser beam, b) size of the cell lattice.
2. Using their prior knowledge, students will be able to make quantitative comparison of the microscopic objects such as crystal cells and atoms with the macroscopic objects such as human hair, centimeter, and meter.
3. Advanced! Students will be able to qualitatively explain why people cannot see and feel atoms, and have to use electron microscopy to make them observable. In this, students should be using the wave concept for electrons and translate their prior knowledge about light wave phenomena into a new framework.

### MATERIALS:

- Ring stands, G-clamps, and test tube holders to hold pencil type laser source;
- Standard He-Ne laser may also be utilized depending on the availability;
- Wooden blocks with the tape could be utilized as alternative laser source holders;
- Laser source (pen type)
- White board or any other flat surface (ordinary wall may suffice);
- Meter sticks, measuring tape, any other type of metric rulers;
- Graph paper, chart paper, markers;
- Digital camera with the tripod;
- Masking tape to fix objects on smooth surfaces and ring-stands.

### **CLASS ACTIVITIES**

Aim on the board: “How can X-rays be useful in studying crystals?”

#### Do now:

During the previous lesson we learned that light has a wave nature. List the phenomena related to visible light that can be easily explained based on the wave model. (5 min)

4-5 students are asked to give their answers and explain how the phenomena they listed can be interpreted as a wave phenomenon. (8-10 min)

Then, students are introduced to the diffraction grating formula (15 min):

$$d \sin \theta = n\lambda \quad (1)$$

which can be used to determine the wavelength  $\lambda$  given that  $d$  is diffraction grating,  $\theta$  is the angle between the 1<sup>st</sup> and the 2<sup>nd</sup> maxima on the screen, and  $n=1, 2, 3, \dots$

For extra-credit, students can be offered to find how this formula is created. This can also be done in class depending on the particular class environment. Students are encouraged to use simple mathematical tools to come up with the diffraction grating equation. Two hints are given to facilitate students thinking processes: i) the distance from the diffraction grating to the screen is much greater

than the width of the individual slit that leads to  $\theta_1 \approx \theta$ ; ii) applying trigonometric function of  $\sin \theta$  should be helpful. In cases when majority of the class has advanced math skills, the discussed exercise can be offered in class as it should not take more than 3-5 minutes to derive the formula.

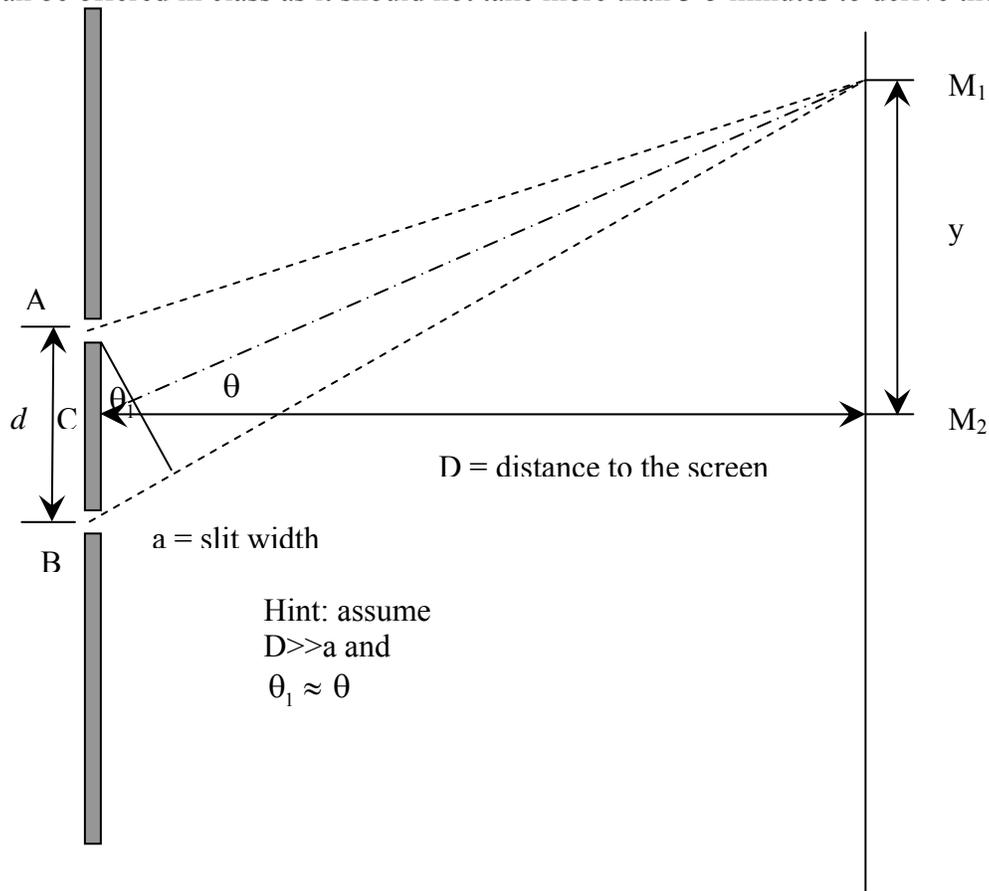


Fig 1 Double slit diffraction and interference.  $\theta = \angle M_1 C M_2$  and  $\theta_1 = \angle B A O$ . One can use concept of  $\sin$  of these angles and the hint from the figure to derive the diffraction grating equation.

**EXERCISE 1:** (30 min)

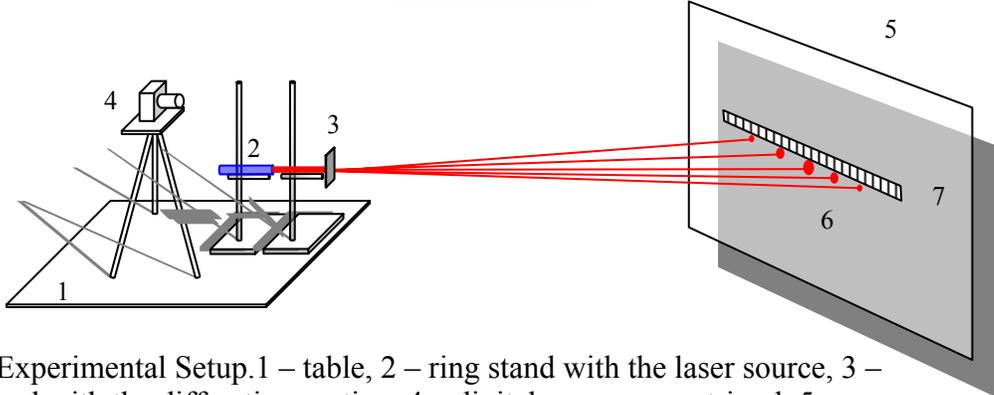


Fig.2. Experimental Setup. 1 – table, 2 – ring stand with the laser source, 3 – ring stand with the diffraction grating, 4 – digital camera on a tripod, 5 – a screen, 6 – observed diffraction and interference pattern, and 7 – a meter stick.

Students in groups of 4 will assemble the experimental setup demonstrated on Fig. 2. Each group will measure the distance from the diffraction grating to the screen or the wall of the laboratory used for the screen. Depending on the number of lines per mm on diffraction grating various distances  $D$  can be appropriate for accurate measurements.

It is expected that students are familiar with the measurement procedures, collecting data, and compiling it in a table. If this is not the case with the particular class a handout (see Appendix I) can be given.

Additional to the diffraction gratings of various groove densities, fine wire meshes can be used to diversify the results. Various laser sources can be used such as semiconductor laser (pen type), school lab laser, and green laser.

In order to document their measurements as well as for their portfolio, students will take digital photos. The digital cameras can also be used to zoom in and measure distance between the fringes more accurately (optional, depends on equipment availability and safety requirements) without being on the way of the laser beam.

Students will find the wavelength of the laser lights and make a summary based on their measurements.

- a) identify different wavelength of different laser sources;
- b) conclude how the diffraction/interference patterns vary from one diffraction grating to another;
- c) predict how the pattern would change depending on the number of lines per mm;
- d) predict how the pattern would change depending on the wavelength of the light source;

Their summary should be close or similar to the following:

- a) wavelengths of different laser sources should be within 10% of the expected value;
- b) diffraction/interference patterns vary by the number of maxima and the way they are arranged:
  - i) fringes become denser with small groove density of diffraction grating lines and vice versa
  - ii) in case of wire meshes, instead of the fringes, there are light spots or maximum spots
  - iii) fringes are parallel to the diffraction grating lines both in standard diffraction gratings and in wire meshes;
- c) as on the number of lines per mm increases and  $d$  decreases fringes or light spots (in case of wire meshes) are more spaced out;
- d) as the green light has the shorter wavelength, the fringes are closer together for the same groove density;

After students have found the wavelengths of the laser light and made a summary, class goes on to discussing these questions:

1. What trend do we observe in the diffraction pattern as the wavelength of light gets shorter?
2. Would we be able to see diffraction pattern in this experiment if the wavelength were 1000 times shorter? (extra: make your estimate of the  $\theta$  between the brightest maxima and the center of the diffraction pattern if the  $\lambda = 10^{-10} \text{ m} = 1 \text{ \AA}$ )
3. If we are unable to produce a visible diffraction pattern with such a short wavelength  $\lambda = 10^{-10} \text{ m} = 1 \text{ \AA}$  (one Angstrom) with the conventional diffraction gratings, what other objects do we know that would work as a grid for short light waves? (By the way, such short light waves are called X-rays or Röntgen ray: <http://en.wikipedia.org/wiki/X-ray>)

Expected answers to the above questions should be close or similar to:

1. As the wavelength of light gets shorter the diffraction picture becomes smaller or shrinks. Distance between fringes becomes smaller and the whole image becomes smaller.
2. We would not be able to see diffraction pattern if the wavelength were 1000 times shorter, because the angle would be  $\sim 1000$  times smaller. The pattern would not be visible, because the size of the spot(s) would be 1000 times smaller.
3. Objects that would work as a grid for short light waves and be able to produce a visible diffraction pattern for  $\lambda = 1\text{\AA}$  would be solids as they have regular structure, order in them. Atoms are arranged in order/lines/stacks in solids and they can be used as diffraction grating for X-rays.

At this point, the 2<sup>nd</sup> exercise is offered. See below.

**EXERCISE 2:** (30 min)

In case of X-ray diffraction, diffraction formula changes due to the changes in the geometry of the measurements. The formula is called Bragg's equation:

$$2d \sin \theta = n\lambda \quad (2)$$

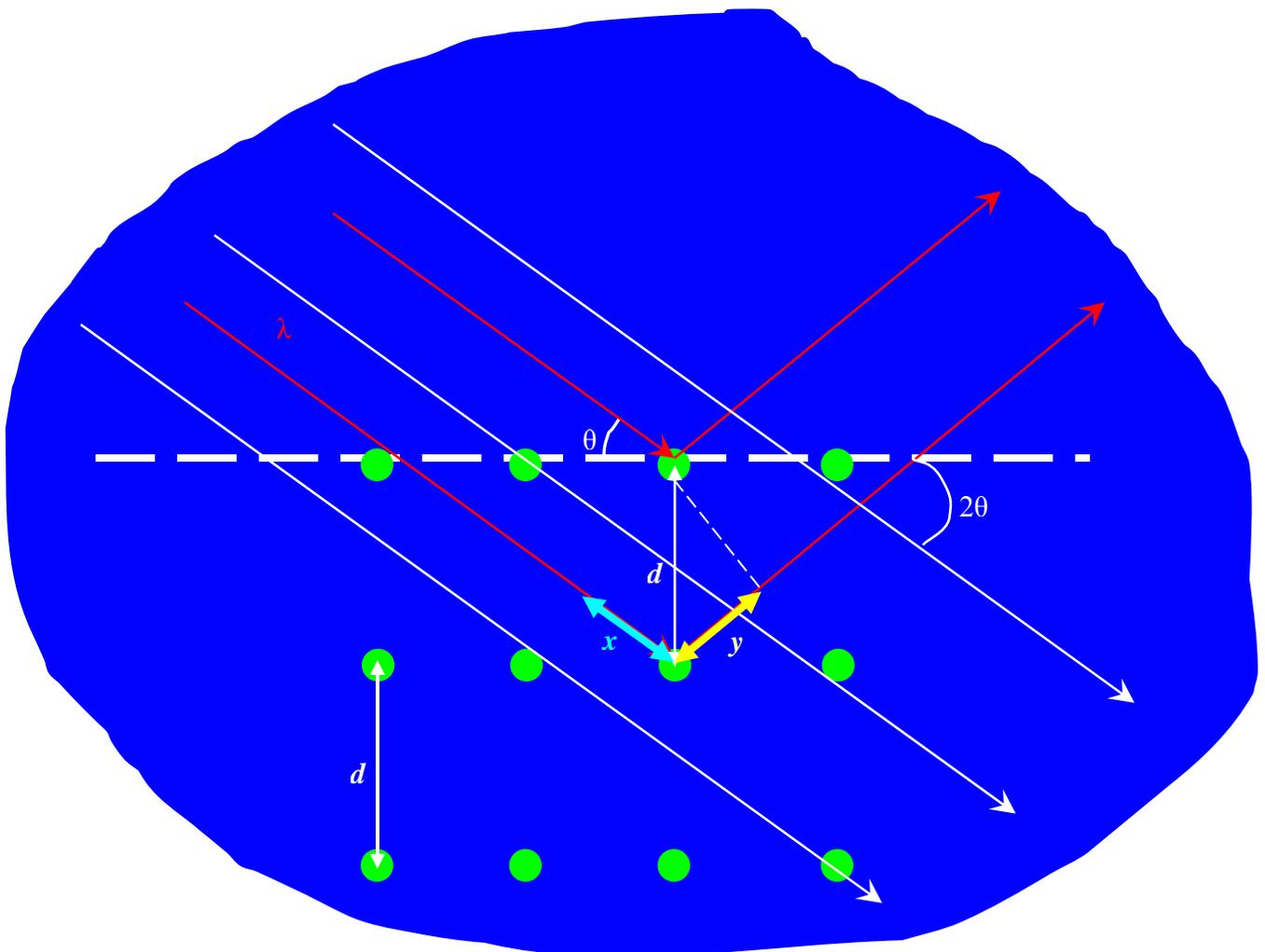
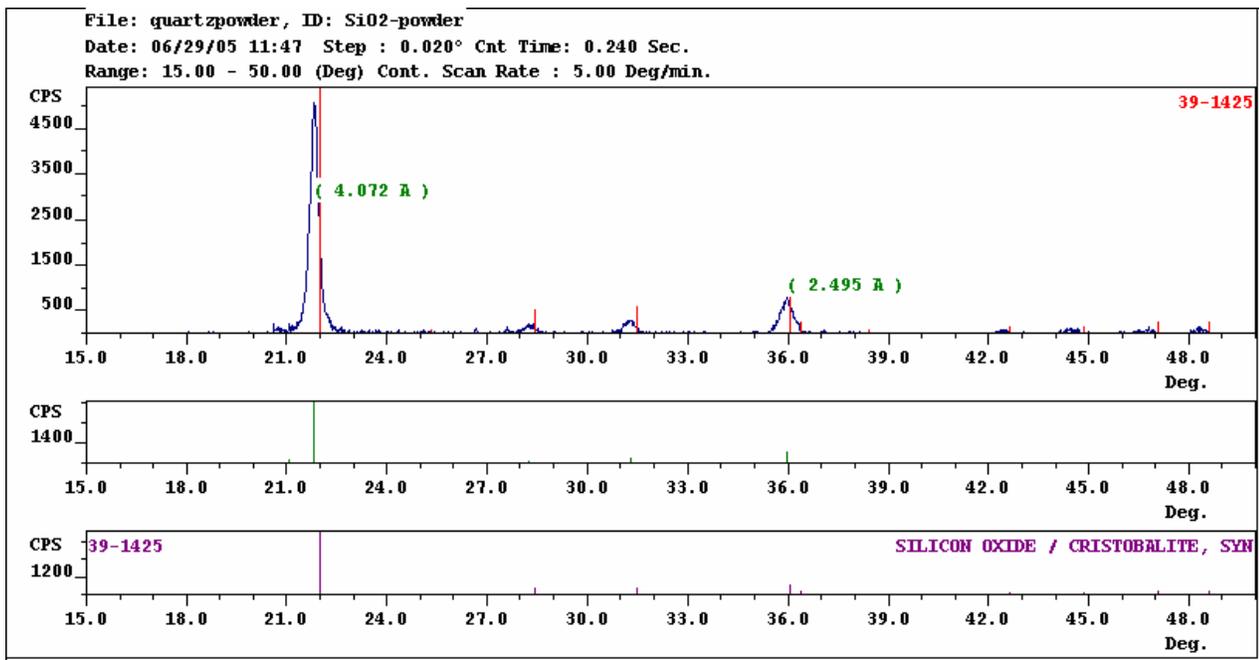


Fig 3. X-Ray Scattered Diffraction Model Geometry

As you can see, it is different from diffraction equation just by a factor of 2. Any student can derive it looking at the Fig 3.

Still not sure how to derive? Use this link:  
[http://www.matter.org.uk/diffraction/geometry/geometry\\_of\\_diffraction\\_braggs\\_law\\_2.htm](http://www.matter.org.uk/diffraction/geometry/geometry_of_diffraction_braggs_law_2.htm)

This is the model students will use to learn from.  
 Given:  $2\theta$  is on the x-axis; d-spacing is given for two peaks.  
 Determine: the wavelength  $\lambda$  of the X-rays.



Answer: wavelength  $\lambda$  for  $\text{Cu } K\alpha_1$  line is  $1.54051 \text{ \AA}$  ( $1.54051 \times 10^{-10} \text{ m}$ )

Students are given worksheets to work on in their groups. They will have to post their answers on chart paper on the board.

Homework is given and explained.

Conclusions are made. (8-10 min)

Conclusions should be reiterations of the previous summaries with more detailed usage of the models.

Additional assessment questions can be asked:

1. How rainbow is produced?
2. Why CD and DVD reflect light differently although they are made of the same material?

The End

## Acknowledgements

I would like to express my gratitude to NSF and CCMR for making RET-2005 happen. Many thanks to CCMR Director Frank DiSalvo, CCMR Associate Director Helen Schember, and CCMR Administrator Lynda Keister.

This work would not have been possible without the continuous support from the Educational Programs Office at the Cornell Center for Materials Research, especially Director of EPO Nev Singhota, EPO Manager Kevin Dilley, and EPO Coordinator Alexa Sabanegh.

Special thanks to:

John Hunt, John Grazul, and Mick Thomas for their patience in making the immense world of light electron microscopy accessible;

Prof. Yuanming Zhang, Anthony Condo Jr., and Prof. Yong Joo for making organic chemistry and chemistry of polymers exciting;

Prof. Maura Weathers for helping make X-ray diffraction applicable in science curriculum in high schools;

Prof. Christopher Umbach, John Sinnott, and Jonathan Shu for introducing Material Science and world of forces both on macro and atomic level;

Two talented men Paul Bishop and Ronald Kemp who make steel and glass ring!

Ken Smith and Sven Pedersen for their support in computing and smooth running of our presentations;

I would like to thank my colleagues: Karrie Frey, Richard Hendrick, Sydney Mendez, Sharon Ramsay, and Youning (Neil) Wang for discussions and ideas they shared.

I would like to thank Pat Viele, Prof. Stephen Sass, Prof. Michael Skvarla, Prof. Robert Thorne, Prof. Matthew Peter Miller, Prof. Hod Lipson, and Gil Toombes for helping teachers make advance science subjects more accessible, fun, and making science education the number one priority.

And last, but not least: I thank the administration of Baruch College Campus High School for continuous and ever growing support they give to science and math teachers.

## Appendix I

### Data collection worksheet

Group #	Diffraction Grating, N lines/mm	d = 1/N, in mm	N (1, 2, or 3...)	Distance from the diffraction grating to the screen, D in mm	Distance from the central maximum to the spot, y in mm	$\sin \theta = \frac{y}{D}$	λ in Å
1.							
2.							
3.							
4.							
5.							

### What Is an Angstrom?

Electromagnetic radiation travels in waves. Scientists use the length of the wave (the distance between peaks) to define the energy of the radiation. Astronomers measure this length in "angstroms," a unit of measure equal to 1 hundred-millionth of a centimeter. It's a convenient shorthand to avoid writing lots of zeroes when talking about the wavelengths of light. In everyday terms, a sheet of paper is approximately 1,000,000 angstroms thick. Visible light covers the range from 4,000 to 8,000 angstroms.

See also: <http://en.wikipedia.org/wiki/Angstrom>

Suggested reading: [http://www.tau.ac.il/~phchlab/experiments/hydrogen/diffraction\\_gratings.htm](http://www.tau.ac.il/~phchlab/experiments/hydrogen/diffraction_gratings.htm))

### EFFICIENCY

The various types of diffraction gratings have different efficiencies versus wavelength characteristics. Classical ruled gratings usually peak with very high efficiency at a certain wavelength and become rapidly less efficient as you deviate from that wavelength. Blazed holographic gratings have similar properties. On the other hand, standard holographic gratings have very little variation in efficiency over the spectral range.

Classically ruled gratings, those ruled mechanically by cutting groove after groove into a substrate with a diamond tool, are characterized by a blaze wavelength and a groove density. The blaze wavelength is the wavelength at which the grating is at its maximum efficiency. Groove density is the number of grooves per mm on the grating surface. The useful range of a grating can be described by the 2/3 - 3/2 rule which simply states that the range of a grating is approximately lower limit 2/3 λ Blaze; upper limit 3/2 λ Blaze Thus for a grating with a blaze of 400 nm, its useful range is 266 to 600 nm. It is not unusual to be able to operate the grating with reasonable efficiency above the "magic" 3/2 value. However, this is not suggested on the short wavelength side below the "magic" 2/3 value.

Links:

1. Vocabulary: <http://amazing-space.stsci.edu/resources/explorations/glossary/amplitude.html>
2. Physics Classroom: <http://www.glenbrook.k12.il.us/gbssci/phys/Class/BBoard.html>
3. Practical physics: <http://www.practicalphysics.org/>
4. Physics for teachers: [http://www.antonine-education.co.uk/Teacher\\_1.htm](http://www.antonine-education.co.uk/Teacher_1.htm)
5. Hyper-physics <http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html>

Appendix II (Students' handouts)  
*Worksheet #1*

**IMPORTANT!**

**Read this before you start. Listen to your teacher's instructions as you conduct the experiment!**

**SAFETY PRECAUTIONS**

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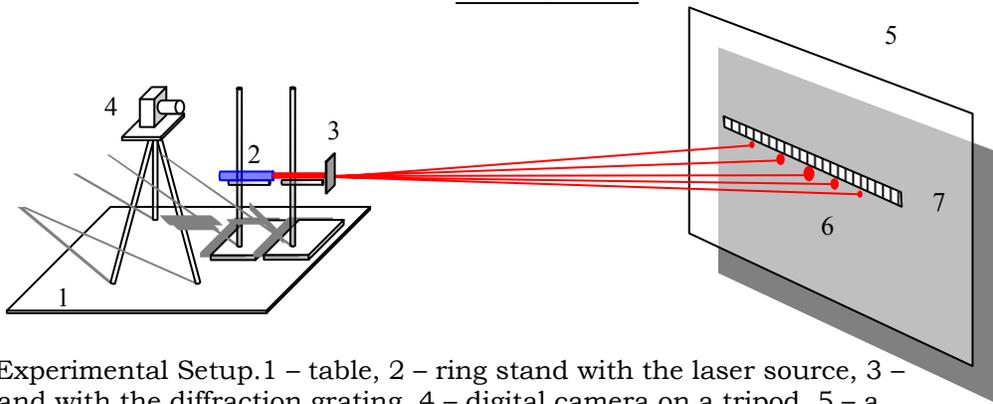


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It is expected that you are familiar with the measurement procedures, collecting data, and compiling it in a table. If you have questions regarding collecting the data, do not hesitate to ask.

Additional to the diffraction gratings of various groove densities, fine wire meshes can be used to diversify the results. Various laser sources can be used such as semiconductor laser (pen type), school lab laser, and green laser.

In order to document their measurements as well as for their portfolio, you will take digital photos. The digital cameras can also be used to zoom in and measure distance between the fringes more accurately (optional, depends on equipment availability and safety requirements) **WITHOUT BEING ON THE WAY OF THE LASER BEAM.**

You will find the wavelength of the laser lights and make a summary based on their measurements.

- e) identify different wavelength of different laser sources;

- f) conclude how the diffraction/interference patterns vary from one diffraction grating to another;
- g) predict how the pattern would change depending on the number of lines per mm;
- h) predict how the pattern would change depending on the wavelength of the light source;

Write your summary below:

e)

f)

g)

h)

After you have found the wavelengths of the laser light and made a summary, go on to discussing these questions:

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4.

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Useful links:

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Additional Information:

### **What Is an Angstrom?**

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### Worksheet #2

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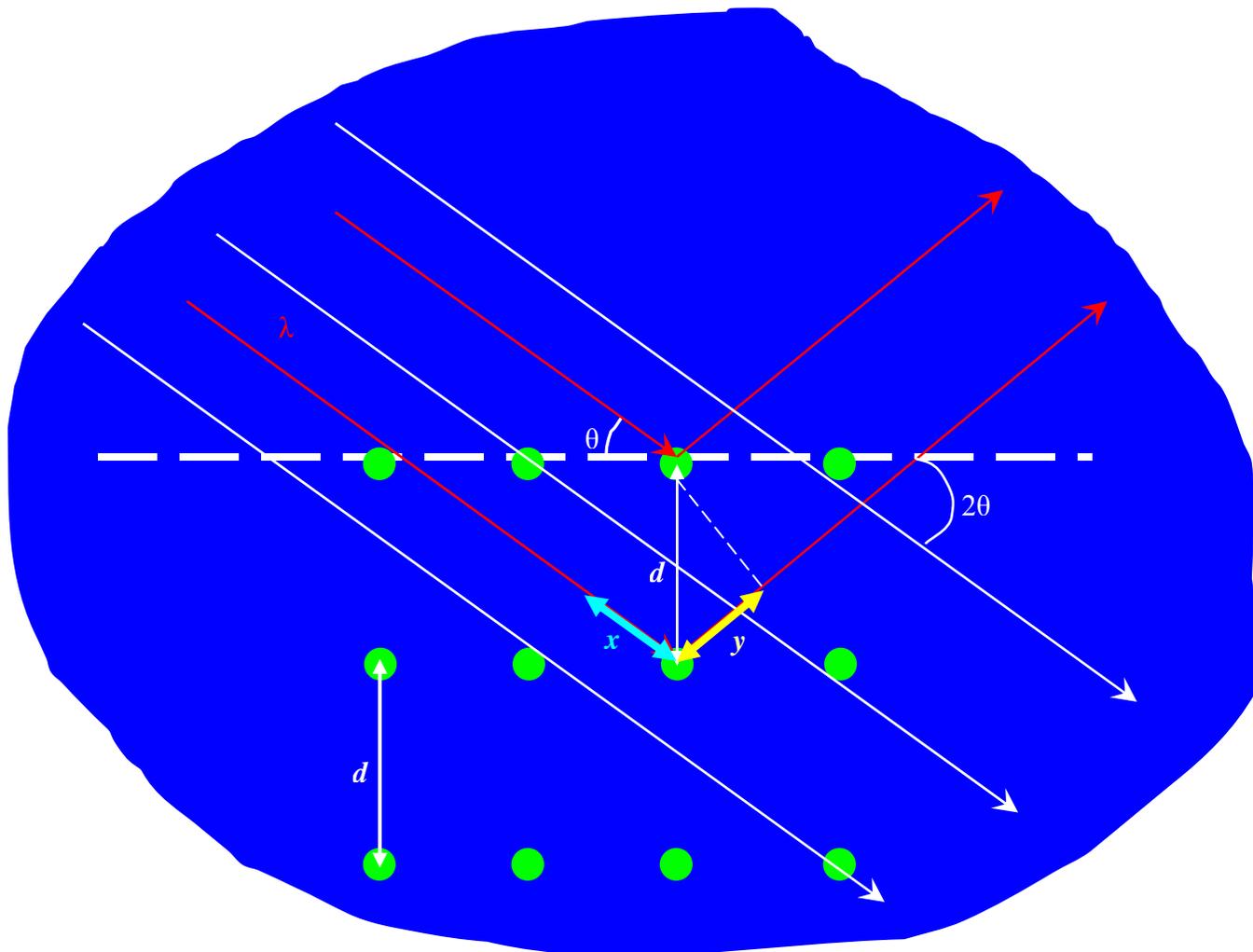


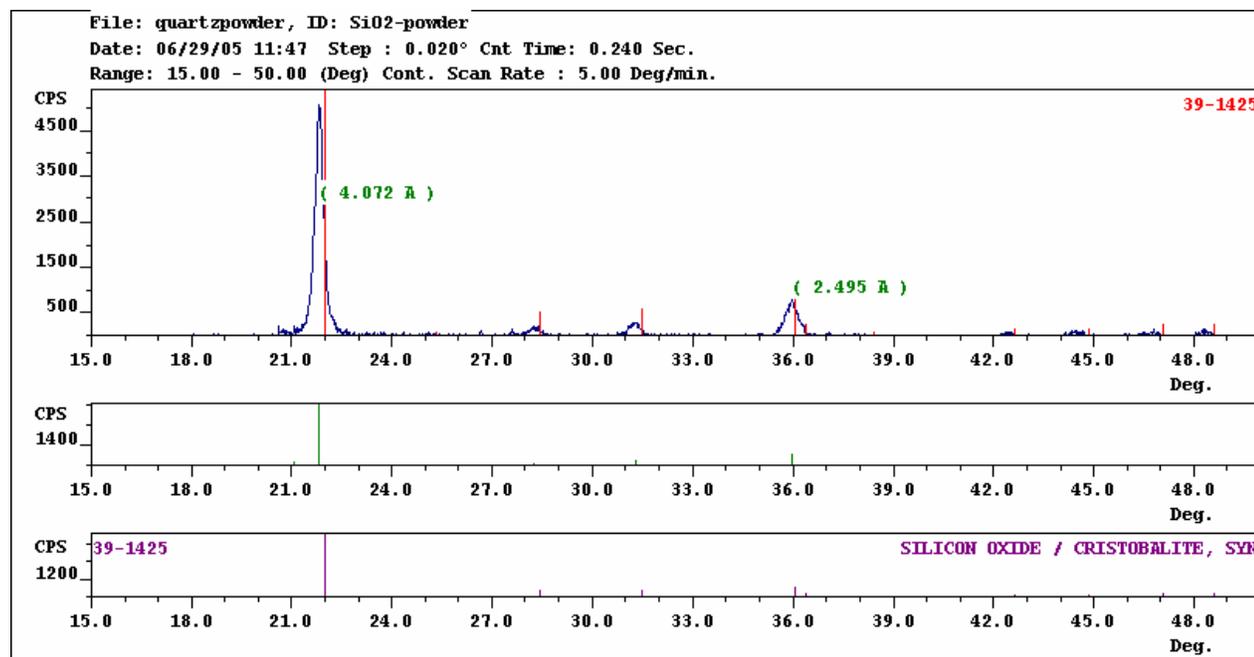
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This is the model students will use to learn from.  
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 Determine: the wavelength  $\lambda$  of the X-rays.



Show your work here:

Answer: wavelength  $\lambda$  for  $\text{Cu } K\alpha_1$  line is \_\_\_\_\_ Å (\_\_\_\_\_  $10^{-10}$  m)

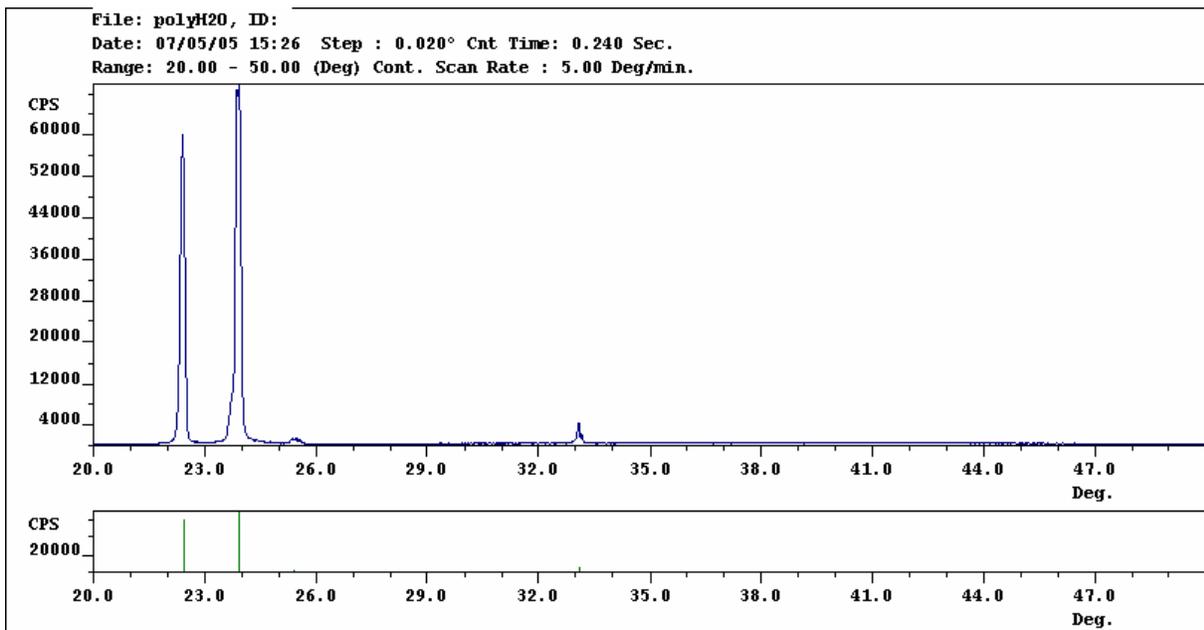
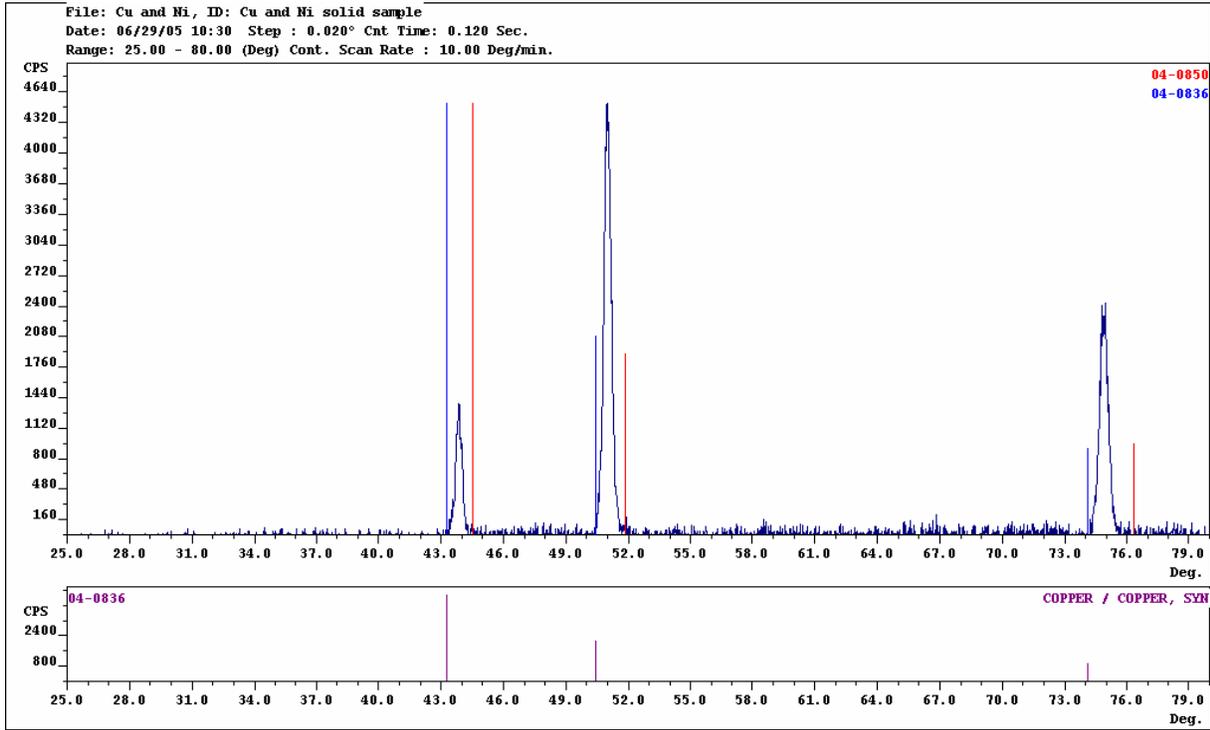
Name \_\_\_\_\_

Class \_\_\_\_\_

From the graphs below, for each solid determine at least 3 different d-spacings.

**Homework:** Why does the same substance have several d's? Your complete explanation with the drawings is due on \_\_\_\_\_.

*Hint: Sketch the atomic structure of simple solid and "shoot" X-rays at different directions*



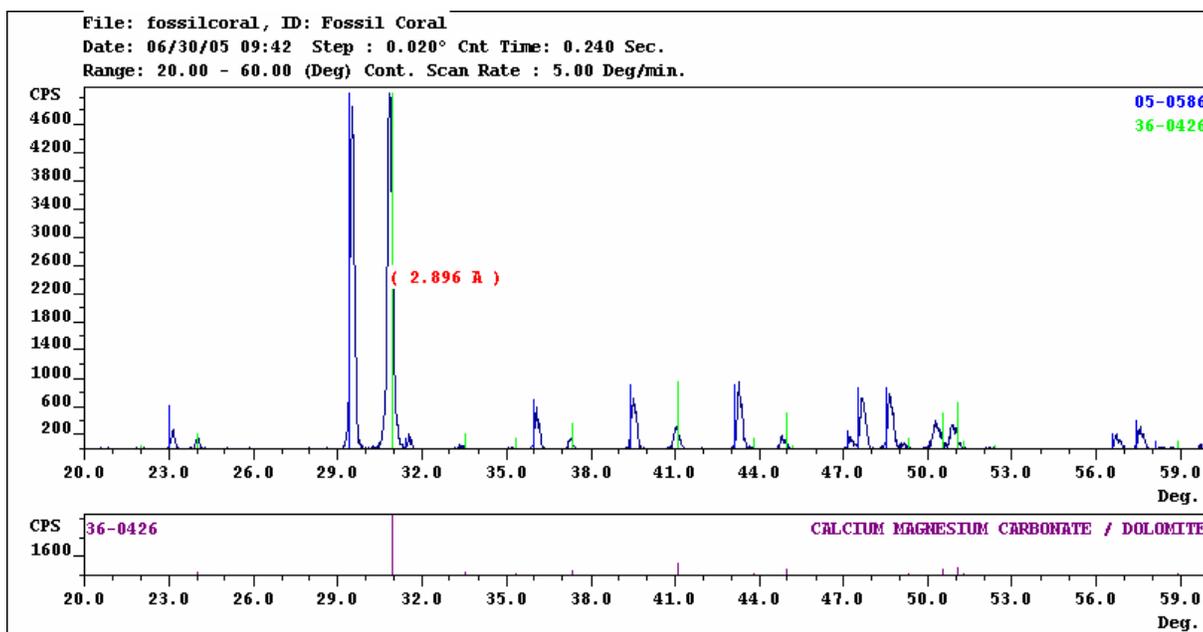
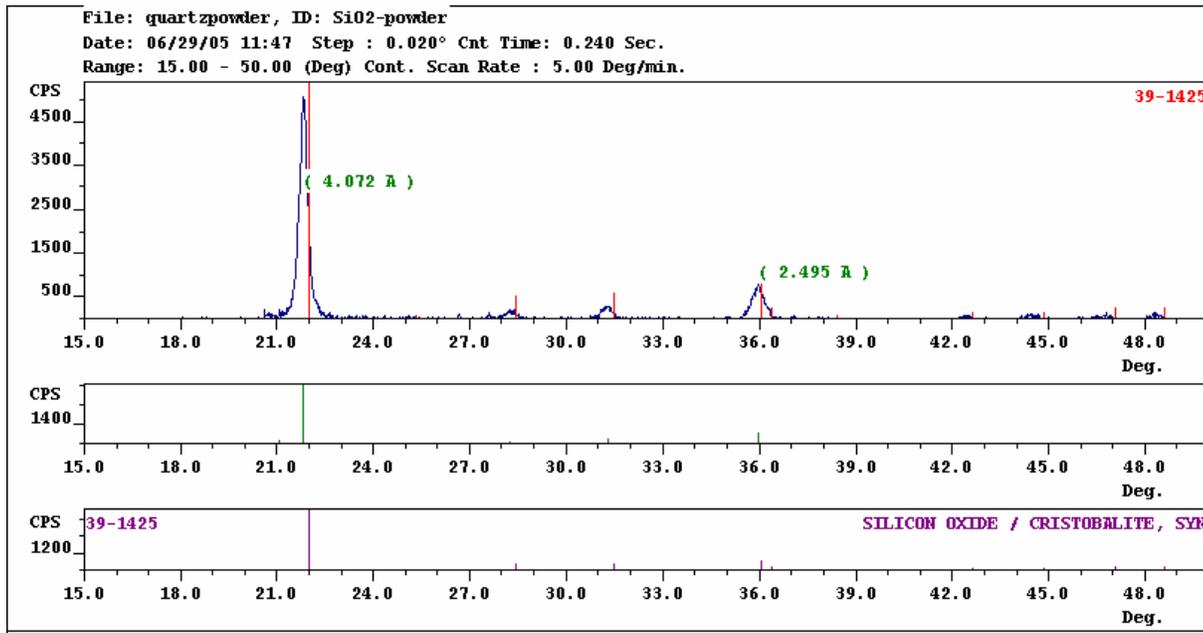
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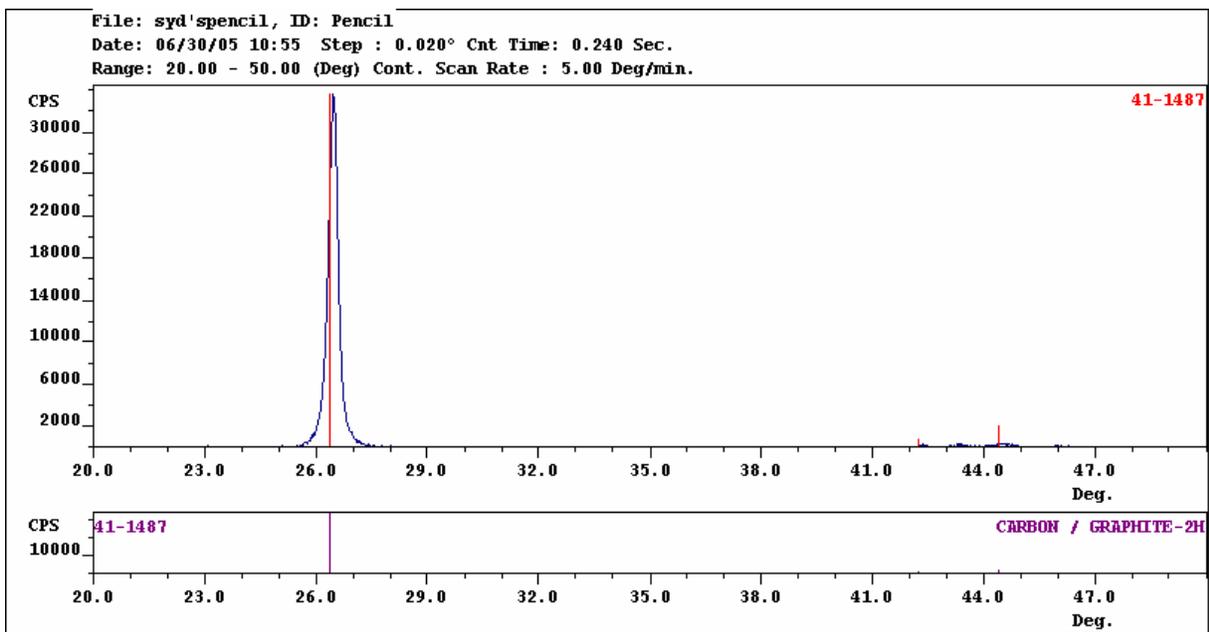
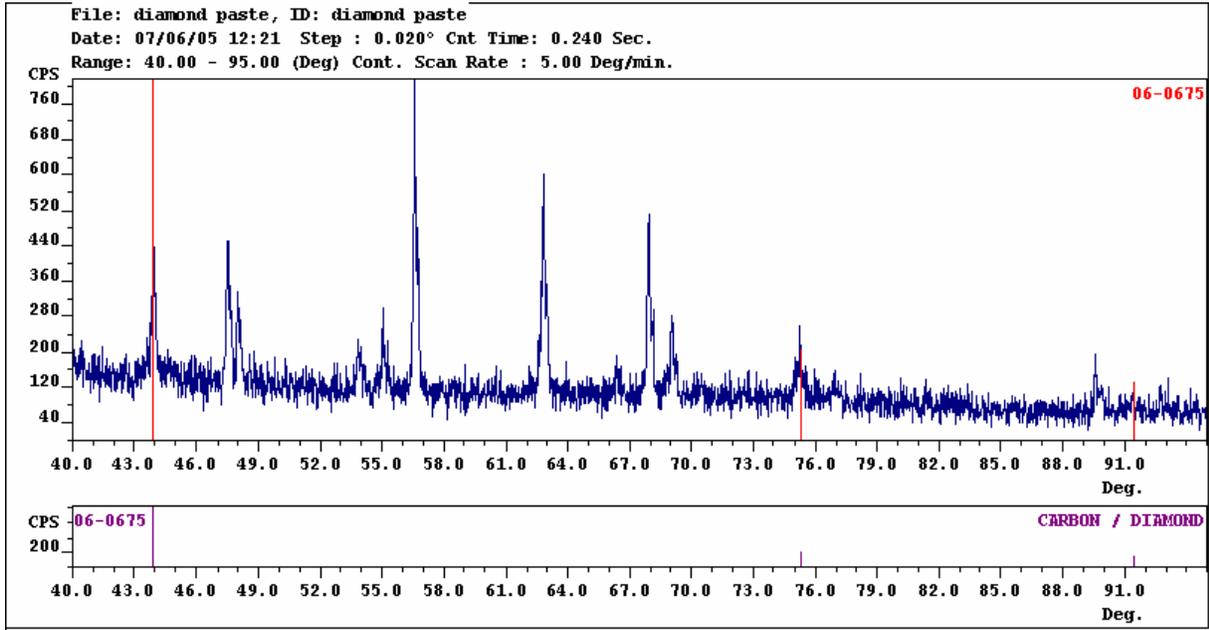
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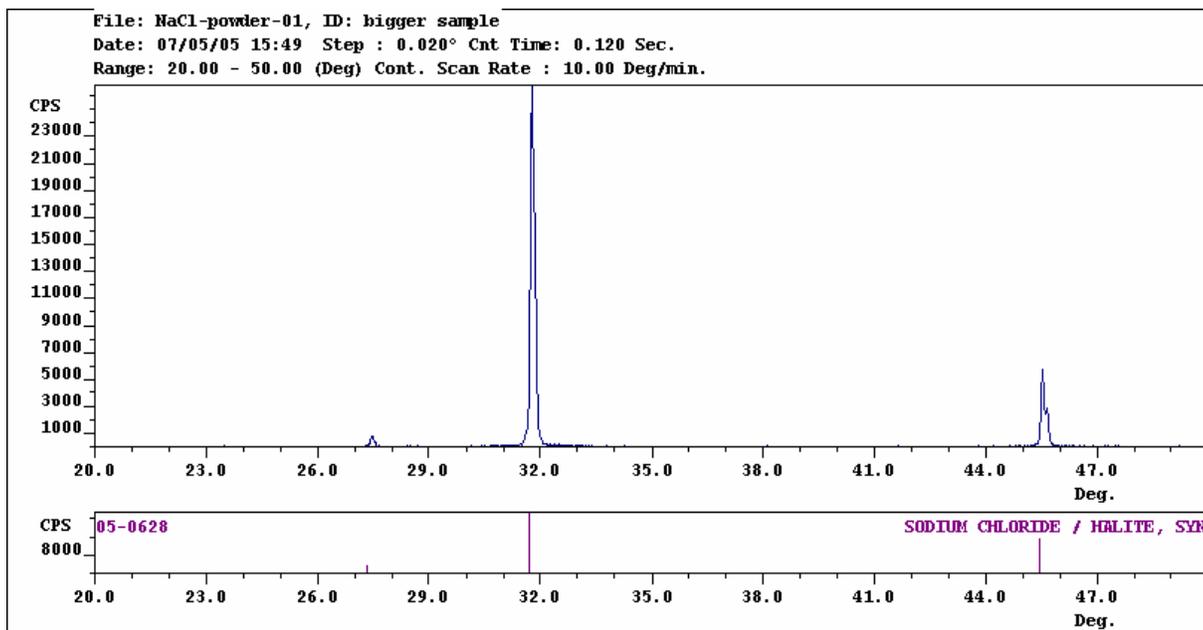
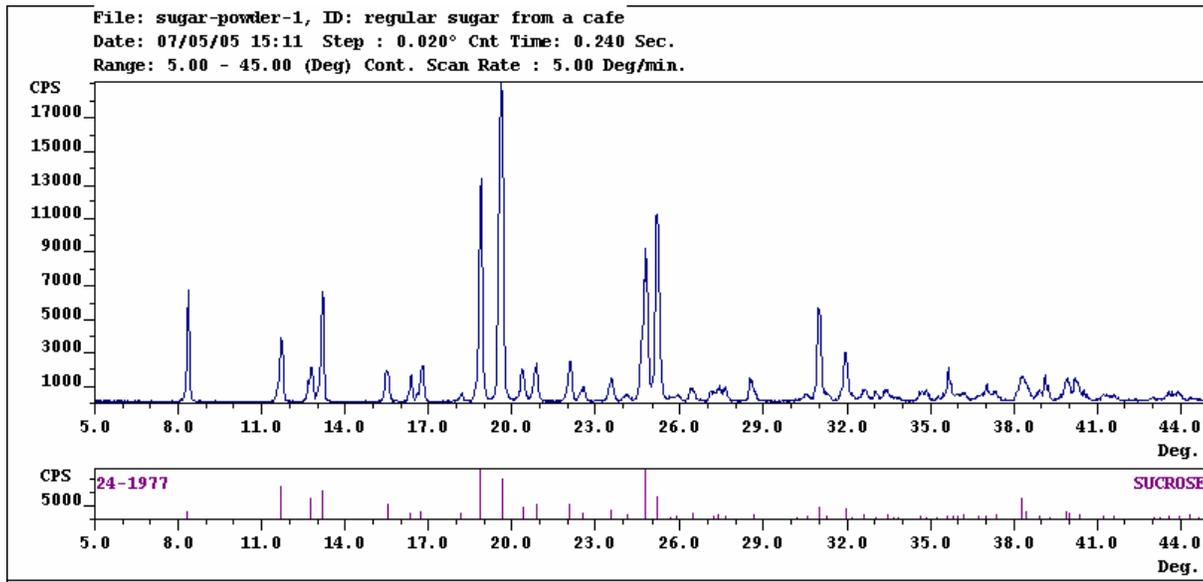
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