INTRODUCTION TO TRIGONOMETRY

A CORNELL CENTER FOR MATERIALS RESEARCH AND NATIONAL SCIENCE FOUNDATION SPONSORED CURRICULUM PROJECT

BY SUSAN DANSKIN

"Introduction to Trigonometry" is a curriculum project created to give eighth grade mathematics students their first taste of trigonometry. Students may begin this unit with absolutely no prior experience with trigonometry. However, it is recommended that they have the following prerequisite skills as they will be integrated into the activities:

- Measuring using metric units
- Measuring angles with a protractor
- Creating scale drawings
- Using similar triangle proportions to find a missing side or angle
- Using Pythagorean Theorem
- Converting ratios to decimals
- Rounding to decimal places
- Comparing and ordering decimals
- Calculating statistics (mean and range)
- Predicting curve shapes based on data interpretation
- Graphing on a number line
- Graphing on the coordinate plane
- Solving algebraic equations

Of course, depending on the level of the students participating, it may be necessary to provide review of some skills as they arise during the activities.

As the title implies this project is only intended to be an introduction to the study of trigonometry. Its focus is governed by the New York State Department of Education's Mathematics Core Curriculum (Intermediate Level) which recommends that students in grades 7 and 8 "find the measure of the sides and angles of a right triangle, using . . . trigonometric ratios." I have included an introductory problem that allows students to demonstrate any prior knowledge as well as provides motivation for the study of trigonometry. A lab component allows the students to discover each of the ratios, the trigonometric tables, and their corresponding curve shapes through exploration of similar triangles. Where applicable, I have provided suggestions for extensions that are not part of the

required curriculum but allow for a more detailed or higher level exploration of the concepts.

Please note that the lessons in this curriculum have been designed for a course that is taught every other day for an 83-minute block. Also, the students in the course for which this project is designed are accustomed to working in cooperative learning groups. Adjustments will need to be made for courses taught in different configurations.

Other resources that you may find useful when preparing a trigonometry unit include:

- Lewis, Geoff. "Sharing Teaching Ideas. The Lost Trigonometry Class and the Hidden Treasure." *The Mathematics Teacher*. 87, no. 1, (January 1994): 19-22.

  A 3-day introductory trigonometry lesson involving attempting to find a hidden treasure in three ways: using a compass and protractor on a scale map, using a magnetic compass and meter sticks to act out the directions, and finally using trigonometry.

- Boyes, G. R. "Trigonometry for Non-Trigonometry Students." *The Mathematics Teacher*. 87, no. 5, (May 1994): 372-375.

  A 3-day introductory trigonometry lesson in which students generate the trigonometric tables using a right triangle in a circle of radius 1 and then work on some practical applications.

- Alvarez, Richard. "How High is the Water Tower?" *The Mathematics Teacher*. 89, no. 4, (April 1995): 274-278.

  Although the mathematics is too complex (pre-calculus) for an introductory trigonometry unit, it describes an interesting activity that could be modified to meet the needs of an introductory course.

- Hersberger, Jim and James O. Farlow. "Tracking Dinosaurs with Trigonometry." *The Mathematics Teacher*. 83, no. 1, (January 1990): 46-50.

  Again, the level of mathematics described is too sophisticated for an introductory course. However, it describes an interesting application of trigonometry that middle school students would be intrigued by.

- For practical application of trigonometry, consult any physics textbook or teacher for problems involving vector analysis.

I would like to thank all of the contributors to this curriculum project. First and foremost, to the Cornell Center for Materials Research my sincere appreciation for the extraordinary opportunity provided by the educational outreach program Research Experience for Teachers. Everyone involved, from the facilities managers who provided us with crash courses in the capabilities of their labs, to the professors and graduate students who shared their research and lesson plans, to the staff of the computer facility who assisted us at every turn, to the outreach coordinator who ran the entire program flawlessly, to my fellow teachers who inspired me with their desire for knowledge, provided for a superior experience. Specifically, I would like to thank Dr. Robert Thorne, Cornell University physics professor, for taking the time out of his busy schedule to discuss trigonometry's applications to physics and Pat Viele, Senior Assistant Librarian at Cornell's Physical Sciences Library, for her assistance in researching the topic. Also, my appreciation goes to the Ithaca City School District, specifically DeWitt Middle School, for fostering creative and alternative approaches to teaching. And finally, thank you to The National Science Foundation for its generous funding of CCMR and all of its outreach initiatives.

Day 1 Lesson Plan

Required Materials

Worksheet “Survival”
Plain or graph paper
Rulers
Protractors
Calculators
Overhead projector, plastic, and pens

- Present the problem to the students by giving each of them a copy of the worksheet “Survival” and reading through it together. Remember that this is the introductory problem for the unit. It is not expected that the students will solve it using trigonometry. Instead they are expected to try to solve it to the best of their ability with the mathematics they already possess.
- Respond to any student questions or comments as they arise until you are confident they understand the problem and the expectations.
- Allow students to work on the problem individually for approximately 10 to 15 minutes thus giving them the opportunity to formulate their own ideas. Students may request any of the available materials as they see fit.
- When you get the feeling the students have progressed as far as possible as individuals and are eager to share their ideas with their group members, allow them to do so. The group’s goal should be not just to come to agreement on the answers to the three questions but to generate as many methods of solution as possible.
- Encourage groups that finish early to try to discover unique methods that other groups may not have produced.
- As groups indicate that they have achieved consensus on the solution and exhausted all of their ideas for methods, provide them with overhead materials so they can prepare a presentation of their group’s results.
- Once all groups have written up their solution and methods, have each group present to the rest of the class using the overhead projector. Act as moderator during the presentations, helping the presenting group to field questions and clarify explanations.

- Following the presentations, survey the class trying to get a feel for which methods tend to be favorite and least favorite and why. I always feel I am better prepared to teach them a new strategy when I am aware of where they are coming from.
- Discuss differences in answers. Are these differences based on incorrect use of mathematics or different levels of accuracy with different methods? Emphasize the importance of exact accuracy in this problem. Remind students that their tribe only wins if it is the first to stop at the exact location of the money. What would be the results of slightly incorrect mileage and what method would be best for avoiding that error? What would be the results of slightly incorrect starting angle and what method would be best for avoiding that error? Emphasize how much even a small error in angle size can put you very far off course.
- Use this discussion of accuracy to motivate the study of trigonometry. By using the trigonometry we can calculate results with far more accuracy than using scale drawings or measuring with a protractor. Demonstrate for the class how trigonometry can be used to answer the "Survival" questions. Be sure to emphasize to the students that they are not expected to learn what you are doing immediately. Instead they will be working their way up to that level of understanding over the course of this unit.
- Introduce students to the language of trigonometry including the parts of the triangle (hypotenuse and legs—adjacent and opposite) and the names of the ratios (sine, cosine, and tangent).
- Allow for in class and at home practice using the vocabulary, labeling the triangle appropriately given the position of the angle θ, and setting up the appropriate ratios.

Extension Opportunity

- Students can explore the results if the tribes travel at different rates. What speeds are required so that the tribes would tie even though Mathematica took the shorter path? so that Mathematica would lose by two hours?

Day 2 Lesson Plan

Required Materials

- Worksheets: Lab 1 Directions, Record Sheet, and Follow-up Questions
- Meter sticks
- Scientific calculators
- Overhead scientific calculator
- Giant floor protractors with string attached (one per group)
  - Options for protractors:
    - Large wooden blackboard versions
    - Handmade oak tag version
    - Small plastic version if other options not feasible
- Large graph paper
- "The Sine Curve"

- Explain that today we are going to complete the first of three labs to explore each of the trigonometric ratios.
- Distribute the worksheets "Lab 1 Directions" and "Lab 1 Record Sheet" and the angle assignments for each group. (Depending on the number of groups, divide the angles 0° through 90° up into even intervals of 5° or 10°.)
- Read through the directions together, demonstrating each step along the way. Emphasize the importance of the accuracy of both the setup and the measurements. Discuss the similarity of the triangles formed by keeping the protractor angle fixed but changing where the string holder stands. Given that similarity, what would they expect to be true of the calculated ratios? Pay particular attention to teaching them how to find and use the trig functions on the calculator so that they will be able to calculate the accepted values of the ratio for each angle.
- Hand out the other equipment necessary for the groups to complete the lab.
- Circulate around the room, assisting groups as necessary.
- As groups finish the trials and complete all required parts of the record sheet, distribute the worksheet "Lab 1 Follow-up Questions".
- So that students can check their predicted curve shapes, create a large ratio vs. angle graph that incorporates the data for each angle investigated during the lab. Have the students take turns plotting the results of each trial, the

mean, and the accepted value. Discuss the general trends of the graph and connect the accepted values to demonstrate these trends better.

- Hand out the copies of "The Sine Curve". Discuss whether or not the points should be connected. If connected, how should the connections be made (straight vs. curved)?

Extension Opportunities

- As an alternative to graphing all of the data by hand, students can enter their data into "Function Probe", "Excel", or some other program capable of producing a graph from data.
- Students can predict and test what happens to the sine curve as the angle involved becomes larger than 90°. Testing could involve conducting the lab at larger angles, using the calculator to generate the ratios, or using a graphing program to plot the function.

Day 3 Lesson Plan

Required Materials

Worksheets: Lab 2 Directions, Record Sheet, and Follow-up Questions
Meter sticks
Scientific calculators
Overhead scientific calculator
Angle templates
Large sheets of paper (4 or 5 per group)
Large graph paper
"The Cosine Curve"

- Repeat the lesson from Day 2 with the following differences: Lab 1 is now Lab 2 and sine is now cosine. It should take significantly less time to introduce the lab because they have been through the similar experience with Lab 1. However, be sure to go through the demonstration of creating and measuring the similar triangles because that part of the lab does differ.

Day 4 Lesson Plan

Required Materials

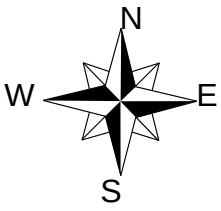
Worksheets: Lab 3 Directions, Record Sheet, and Follow-up Questions
Meter sticks
Protractor
Scientific calculators
Overhead scientific calculator
Meter sticks with weighted strings (one per group)
Large graph paper
"The Tangent Curve"

- Again, repeat the lesson plan from Day 1 with the appropriate worksheet and vocabulary changes. Be sure to go through the demonstration of using the meter stick with weighted string to create and measure similar triangles for the four trials.

Intro to Trigonometry Name_____
"Survival"

You have been selected to participate in a new and improved television show called "Survival". Your tribe, Mathematica, will be competing against the tribe named Dunceheadia for the prize money of $1,000,000. All you have to do to win is be the first tribe to make its way through the jungle and stop at the exact location where the money is buried.

On the day the competition is to begin, the cameras are rolling as the two tribes gather at the starting point and Tamara Trigonometry, the show's host, describes the challenge:



"$1,000,000 in cash awaits the winner of today's challenge. All you have to do to claim the prize money is be the tribe who stops at its exact location first. Directions to the location are simple: from the starting point, head due east for 28 miles then turn 90° and head due north for 21 miles. No, ladies and gentlemen, "Survival" is not for wimps! Good luck Dunceheadia, Mathematica! On your mark, get set, GO!"

With that Tribe Dunceheadia sets off into the jungle following the directions exactly. They will travel through the thick jungle at a steady rate of 1 mph.

Tribe Mathematica, your tribe, is not in such a hurry to get started. Instead, you take the time to have a strategy session. You know that your tribe moves at the same rate of speed as your opponents. Therefore, it would be foolish to follow the same trail because at best the results would be a tie and another competition would have to be staged. You decide to take an alternate route, the shortest distance to the remote location where the money awaits you.

The strategy session is devoted to answering three important questions:

1) At what angle should we head to insure we are aimed straight at the money's hidden location?

2) After how many miles of walking should we stop to insure that we are directly over the money's buried location?

3) If it takes us 1 hour to strategize and we travel through the jungle undergrowth at a rate of 1 mph, how much time can we use to stop for eating and sleeping and still beat Tribe Dunceheadia by 2 hours?

Each member of your tribe should strategize individually at first. Then all members of your tribe will work together to produce your best tribe strategy. Remember that when the competition is completed the news media will want a report on your strategy including your methods for finding the answers to the three important questions

Intro to Trigonometry
Lab 1 Directions

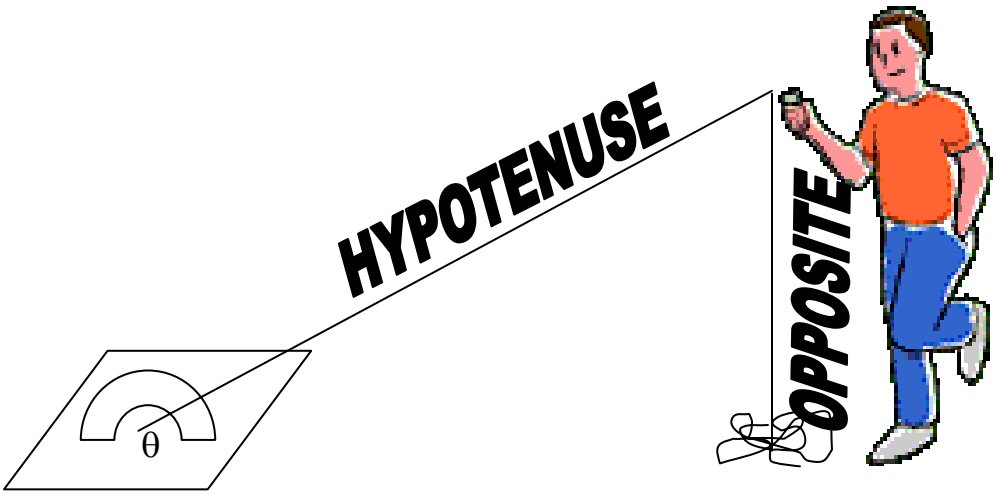
Name_____
Date_____Period_____

Required Materials

Lab 1 Directions	Lab 1 Record Sheet
Meter Sticks	Scientific Calculator
Pencil	Giant Floor Protractor with String

Directions

1) Make sure your group has the required materials.
2) On your record sheet, fill in the angle size column with the values assigned to your group. Depending on how many angles you are assigned, you may not need all of the rows provided in the table.
3) Assign each person in your group one of the following jobs: angle checker, string holder, and 2 measurers. Make sure you rotate these jobs occasionally so that each person gets to perform each job.
4) Conduct the trials for each angle by setting up the protractor and string as pictured here. Make sure the string holder stands in a different place for each trial. No, you do not have to stand on one foot!!



5) Measure, using centimeters, the lengths of string required to complete the opp : hyp ratio for each trial.
6) When you have completed all of the trials, convert each ratio to a decimal rounded to the nearest ten-thousandths (4 decimal places).
7) Calculate the mean (rounded to 4 decimal places) of the four trials for each angle size.
8) Find the accepted value (rounded to 4 decimal places) of the ratio using a scientific calculator.
9) When you have completed the table, begin work on Lab 1 Follow-up Questions.

Intro to Trigonometry
Lab 2 Directions

Name_____
Date_____Period_____

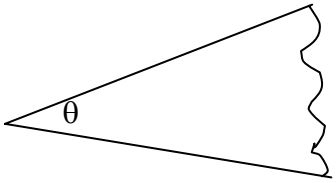
Required Materials

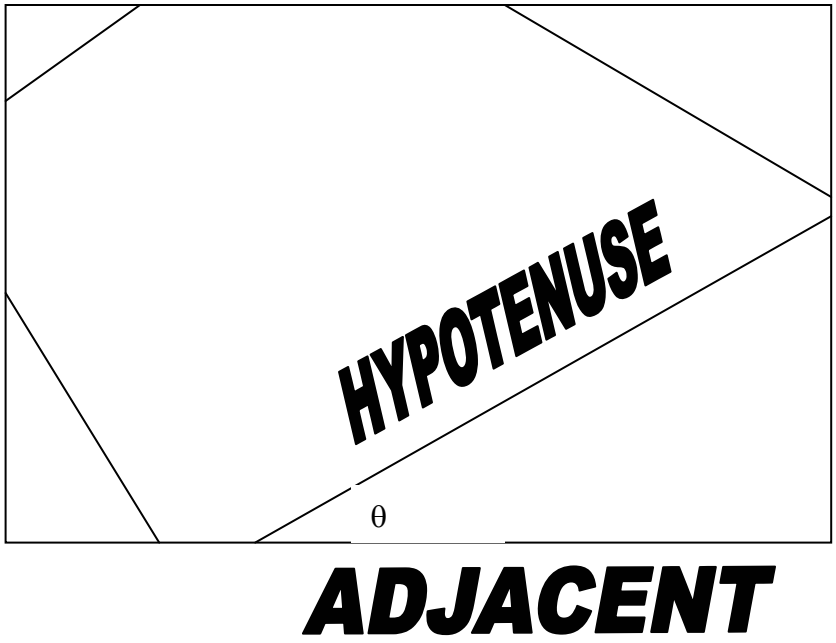
- Lab 2 Directions
- Meter Sticks
- Pencil
- Angle templates
- Lab 2 Record Sheet
- Scientific Calculator
- 4 or 5 large sheets of paper

Directions

1) Make sure your group has the required materials.
2) On your record sheet, fill in the angle size column with the values from your angle templates. Depending on how many angles you are assigned, you may not need all of the rows provided in the table.
3) For each angle template, create 4 similar but non-congruent triangles as pictured here. Cut out the triangles so that you can use them for your 4 trials.

Angle Template





4) Measure, in centimeters, the lengths of the triangle required to complete the adj : hyp ratio for each trial.
5) When you have completed all of the trials, convert each ratio to a decimal rounded to the nearest ten-thousandths (4 decimal places).
6) Calculate the mean (rounded to 4 decimal places) of the four trials for each angle size.
7) Find the accepted value (rounded to 4 decimal places) of the ratio using a scientific calculator.
8) When you have completed the table, begin work on Lab 2 Follow-up Questions.

Intro to Trigonometry
Lab 3 Directions

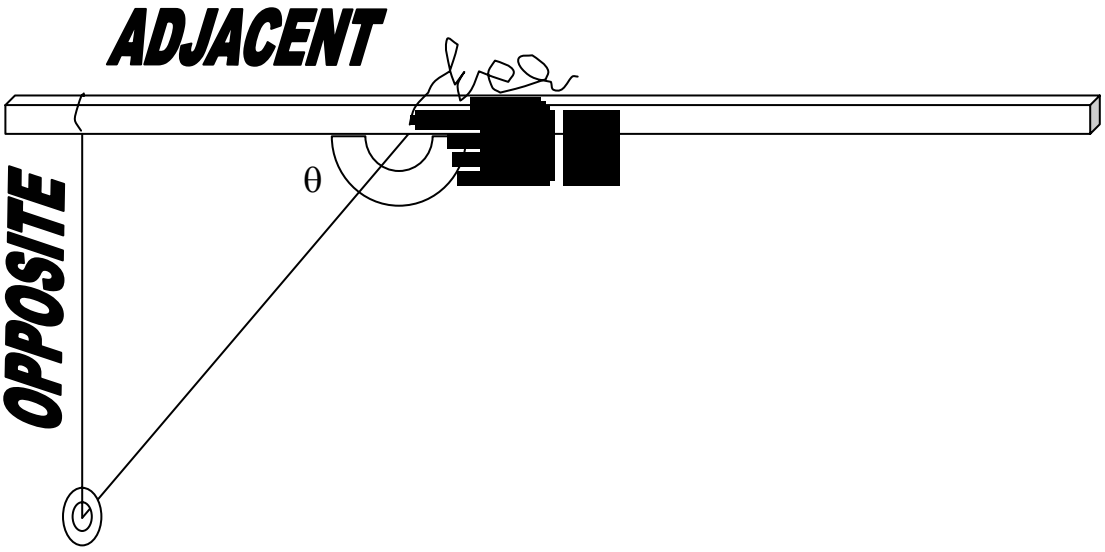
Name_____
Date_____Period_____

Required Materials

- Lab 3 Directions
- Meter Sticks
- Protractor
- Pencil
- Lab 3 Record Sheet
- Scientific Calculator
- Meter Stick with Weighted String

Directions

1) Make sure your group has the required materials.
2) On your record sheet, fill in the angle size column with the values assigned to your group. Depending on how many angles you are assigned, you may not need all of the rows provided in the table.
3) Assign each person in your group one of the following jobs: angle checker, string holder, and 2 measurers. Make sure you rotate these jobs occasionally so that each person gets to perform each job.
4) Conduct the trials for each angle by setting up the system pictured here. Make sure the weight hangs at a different length for each trial.



5) Measure, using centimeters, the lengths of string required to complete the opp : adj ratio for each trial.
6) When you have completed all of the trials, convert each ratio to a decimal rounded to the nearest ten-thousandths (4 decimal places).
7) Calculate the mean (rounded to 4 decimal places) of the four trials for each angle size.
8) Find the accepted value (rounded to 4 decimal places) of the ratio using a scientific calculator.
9) When you have completed the table, begin work on Lab 3 Follow-up Questions

Intro to Trigonometry
Lab 1 Record Sheet

Name_____
Date_____Period_____

Angle Size (θ)	Trial 1 $\frac{\text{Opp}}{\text{Hyp}}$ = decimal to 4 places	Trial 2 $\frac{\text{Opp}}{\text{Hyp}}$ = decimal to 4 places	Trial 3 $\frac{\text{Opp}}{\text{Hyp}}$ = decimal to 4 places	Trial 4 $\frac{\text{Opp}}{\text{Hyp}}$ = decimal to 4 places	Average (Mean to 4 places)	Accepted value of opp : hyp ratio to 4 places
	---------- =	---------- =	---------- =	---------- =		
	---------- =	---------- =	---------- =	---------- =		
	---------- =	---------- =	---------- =	---------- =		
	---------- =	---------- =	---------- =	---------- =		
	---------- =	---------- =	---------- =	---------- =		

The ratio we examined in this lab (opp : hyp) is referred to by mathematicians and scientists as the

_____ ratio.

Intro to Trigonometry
Lab 2 Record Sheet

Name_____
Date_____Period_____

Angle Size (θ)	Trial 1 Adj = decimal to 4 places Hyp	Trial 2 Adj = decimal to 4 places Hyp	Trial 3 Adj = decimal to 4 places Hyp	Trial 4 Adj = decimal to 4 places Hyp	Average (Mean to 4 places)	Accepted value of adj : hyp ratio to 4 places
	---------- =	---------- =	---------- =	---------- =		
	---------- =	---------- =	---------- =	---------- =		
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	---------- =	---------- =	---------- =	---------- =		

The ratio we examined in this lab (adj : hyp) is referred to by mathematicians and scientists as the

_____ ratio.

Intro to Trigonometry
Lab 3 Record Sheet

Name_____
Date_____Period_____

Angle Size (θ)	Trial 1 $\frac{\text{Opp}}{\text{Adj}}$ = decimal to 4 places	Trial 2 $\frac{\text{Opp}}{\text{Adj}}$ = decimal to 4 places	Trial 3 $\frac{\text{Opp}}{\text{Adj}}$ = decimal to 4 places	Trial 4 $\frac{\text{Opp}}{\text{Adj}}$ = decimal to 4 places	Average (Mean to 4 places)	Accepted value of opp : adj ratio to 4 places
	---------- =	---------- =	---------- =	---------- =		
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The ratio we examined in this lab (opp : adj) is referred to by mathematicians and scientists as the

_____ ratio.

Intro to Trigonometry
Lab 1 Follow-up Questions

Name_____
Date_____Period_____

1) Give some possible explanations for why your calculated values for the sine ratio differ from the accepted values.

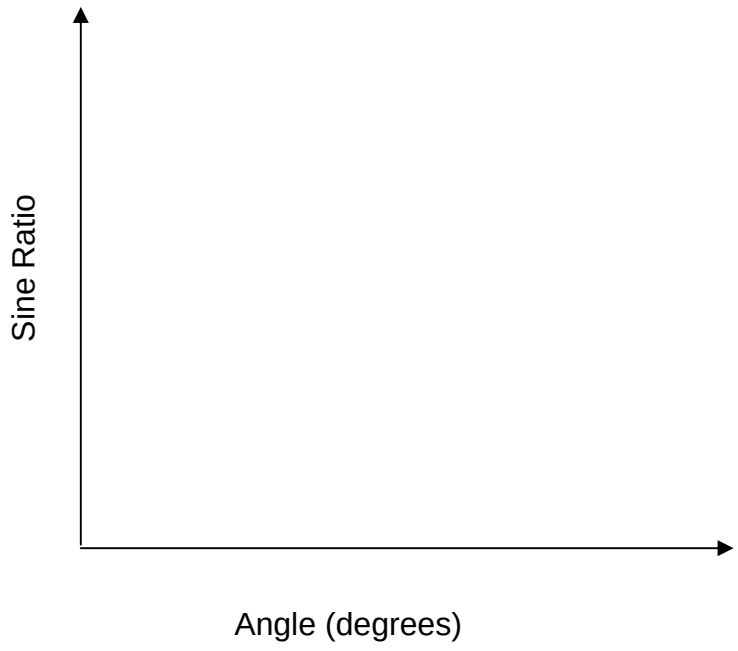
2) Make at least 3 observations regarding the accepted sine ratio. (Hint: You might want to consider the range of data as well as the trend of what happens to this ratio as the angle size increases.)

   a)

   b)

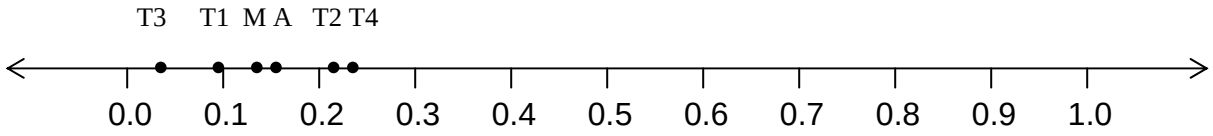
   c)

3) Based on your observations in #2, make a sketch of your predicted curve shape for the sine ratio on the given axes

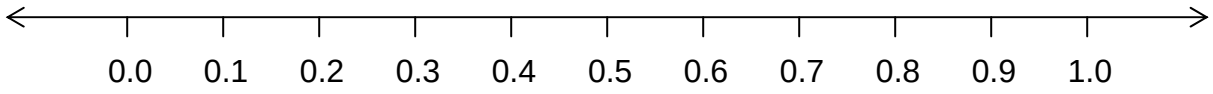


4) For each of the angles at which you performed the lab activity; graph and label the sine ratio for each trial (T1-T4), the mean (M), and the accepted (A).

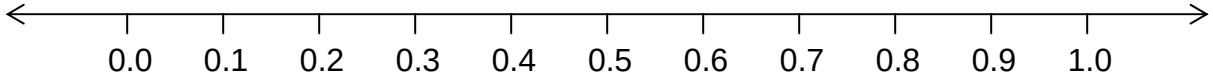
Example for imaginary data:



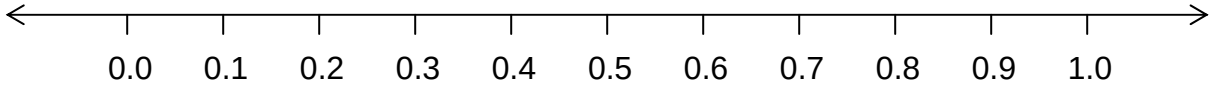
a) Angle Size (θ) = _____



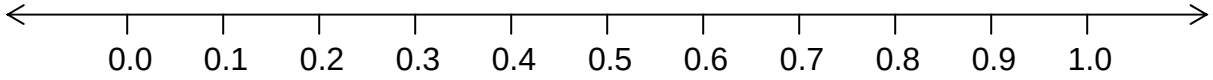
b) Angle Size (θ) = _____



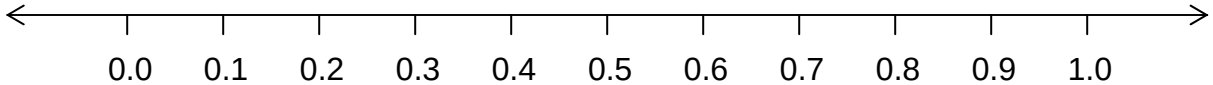
c) Angle Size (θ) = _____



d) Angle Size (θ) = _____



e) Angle Size (θ) = _____



Intro to Trigonometry
Lab 2 Follow-up Questions

Name_____
Date_____Period_____

1) Give some possible explanations for why your calculated values for the cosine ratio differ from the accepted values.

2) Make at least 3 observations regarding the accepted cosine ratio. (Hint: You might want to consider the range of data as well as the trend of what happens to this ratio as the angle size increases.)

   a)

   b)

   c)

3) Based on your observations in #2, make a sketch of your predicted curve shape for the cosine ratio on the given axes