PHYSICS 653 (Fall 2003) LIST OF LECTURES -- OUTLINE

Outline for each lecture, also index to the lecture-related handouts.
(Symbols used: @ typed handout # handwritten handout)

Homework problems and teaser questions are listed too, at the end!

UNIT I. RANDOM WALKS

1.1 Random walks in 1D

A. Overview: why walks?
walks in time; in a state space; or in
space as configuration of string-like objects
Diffusion w/traps - e.g. biol. receptors, enzymes
B. Defining models
defn's "random process" "Markov process"
C. Diffusion equation {deriv. from discrete}
D. Return to the origin (mean no. of returns)
D'. [handwaving picture of dimension dependence]
<Parenthetical 1.1X, 9/6/01: Walk w/ persistent velocity v(t)
which switches occasionally at random from +v to -v.>
E. Distribution of survival times (1D w/trap/method of images)
F. Survival in a bounded interval (q/Fourier modes)
(brief summary of fractal diffusion)

@ suppl., "Motile Behavior of Bacteria" by Howard C. Berg (Jan 00 Phys. Today)
@ T.Q. followups
I (1.1-1.2) Interacting walkers -- hard core exclusion
II (1.1) more about interacting walkers -- annihilation/diffusion problem
III (1.1) motivation for the diffusion Greens function
(1.1F') diffusion with a set of fixed traps

1.2 Dimensionality and continuum diffusion

preview: Adam & Delbruck, 1968, formula for trapping time in d=1,2,3.
@ {dimensional reduction and trapping time: fig. of photochemical
[chlorophyll] reaction center, from Alberts et al, Molecular
Biology of the Cell, 1989 ed.]
A. Set-up : concentration c(x), flow current, boundary conditions
electrostatic analogy
B. Fate of diffuser: rel. prob. to hit each trap (or to infinity)
1) planar (1D) 2) concentric spherical
C. mean lifetime (time till capture)
= integrated c(x)/trapping current (deriv'n)
1) planar (??) 2) concentric spherical
D. lifetime in spherical geometry
< omitted: arrays of chemoreceptors (Berg & Purcell)>

1.3 Master equation (dynamics of discrete states)
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@ handout [titled Lec. 21, from P 653/2000]

A. Examples
1) channel protein, micropipette, telegraph noise
2) molecular motors (kinesin/microtubules)
3) point-contact junction
4) interstitial atoms diffusing in a crystal
5) Monte Carlo simulation of Ising models
B. Set-up
Markov process, discrete vs. continuous time ,
Master eqn in matrix form, l. and r. eigenv.
Frobenius theorem (=> unique steady state)
C. Detailed balance
1) "irrotational" prob. flow in state space
2) statistical time-reversal symmetry
<Omitted Ising model dynamics:
Metropolis, heat-bath/Glauber; spin-conserving>
D. Rel. to experiments (with membrane channels)
1) dwell-time dist. 2) [skipped?] corr. fcn <I(0)I(t)>
<Time correl fcns by diagonalizing transition matrix
2 (#) correlation fcns; example -- random walk >

UNIT II SPIN MODELS AND CRITICAL PHENOMENA
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2.1. Models and symmetry breaking
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(grand overview)
A. Lattice models, discrete v. continuous symmetry
1) Lattices (pictures) #
2) n-vector spin models (table) #
Realizations of models (table) #
B. Symmetry breaking
1) order of limits of M(h) (h -> 0 and N -> infinity)
2) continuous vs. discrete: stiffness
B’. lower critical dimensionality (and elasticity)
C. 1D Ising model via kinks
D. Fluctuations and correlations
1) Correlation function
subtraction of LRO, T dependence, meaning
<correlation length -- OMITTED??>
2) fluctuations in equil.
3) susceptibility sum rule
@ 2.1 followup -- handout on symmetry breaking

2.2 Criticality
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@ "Lec 4. Criticality" from 653/2000
A. Critical exponents
1. Sp ht C(T) 2. Spont mag 3. Susc. 4. Correlation fcn
(A) Table of exponents (table and notes) @
B. Susceptibility sum rule [sum of G(r)] (important! skipped?)
C. Universality, dimension dependence
lower/upper critical dim; systematic (n,d)

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2.3 Mean-field theory
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   A. Overview
       Why important
       Sitewise vs. Landau theory forms of MFT
   B. Landau-Ginzburg theory
        1) <--> micro. model (coarse graining)
        2) L-G functional (via symmetry properties)
   C. Critical exponents from Landau theory
         alpha, beta, gamma, delta
   D. # Scaling form
     {Note Ginzburg criterion, mentioned in class 10/2}

# Special handout (1995 Lec. 9) "Scaling and exponent relations"

2.4 Correlations in Mean field theory
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   A. Correlation function <--> Fourier transform
   B. Fluctuations from Landau-Ginzburg
   C. Results
           1) rescaling, 2) G(R),  3) exponents nu, eta
  #D. Ginzburg criterion

2.5 Renormalization group basics
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   @ = "Lec 10" from 653/2000
   A. The basic idea
    (@) Handout: pictures from Yeomans book
     1. Setup:
     2. Coarse graining
     3. "Partial trace" over weights
     4. New Hamiltonian H, iteration
   B. R.G. identities
   C. R.G. flows -- topology terminology
     Handout (#) sketches of flows + names
     1  fixed points,  2 xi along flow  3. basin/separatrix,
     4. critical f.p., physical curve  5. f.p. w/2-parameter flow
         kinds of fixed point (table #)

2.6. Renormalization group flows and exponents
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   @="Lec 11 outline" from 653/2000
   See also @="Lec 9, Scaling" handout (incl. exponent relations)
   A. recap d=1 Ising model (review)
       1D Ising decimation R.G. #
   B. (Why) Exact decimation RG fails in d>1 #
   C. extracting critical behavior: one-param. flow
     (input: the RG identities)
   D. Two-parameter scaling
   E. Irrelevance
   F. Crossovers (table) #

Added brief:
   * Real-space perturbative R.G. (from 653/2000/lec 13)
   * Momentum-space R.G. (from 653/2000/lec 14)
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UNIT III. STOCHASTIC DIFFERENTIAL EQUATIONS
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3.1 Introduction to stochastic dynamics  [real variable] (# outl.)
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     @ "Stochastic processes in physics", extract from van Kampen (1992 ed.)

     A. Overview  of this unit
 handout from N. G. van Kampen book (*separate*)
     B. Random processes
           1. Correlation fcn G(t)
           2. Correlation time
     C. Noise spectrum -- Fourier (#)
            1) Wiener-Khintchine theorem
            2) Examples (1/f, white noise)
            3) Dynamic structure factor S(q,omega)
     D. Response function: X(t)
                 effect decays (system returns to unique equil. state)
     E. Kramers-Kronig (brief)



3.2 Stochastic differential equations
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      A. "Continuous time" random walk => diffusion eqn.
      B. evolution equation of P(phi)
               dP/dt (drift and noise terms)
               position variable , biased r. walk => "Smoluchowski eqn"
               velocity variable, Langevin eqn    => "Fokker-Planck eqn"
         Stochastic D.E.’s (table) #
     C. Linear response
        steady state is Boltzmann dist.; detailed bal => prob. current J=0.
        (3.2C) "Alternate derivation of w(phi)"(skipped in lec.)
     D. Johnson noise [was done belatedly 11/6/03]



3.3 Fluctuation-dissipation relation I: Einstein relation
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     {building one random process atop another}
     A. Einstein relation
        {@ 2 Figures from "Laser Tweezers in Cell Biology, ed. M. P. Sheetz}
     B. Discussion
           1) Motional narrowing
           2) Why linear response even beyond equilib. fluct. scale?
     C. Kernels (#)
           1) relaxation-time approxn
           2) long-time tails (a. response, b. correl. function)

3.4 Fluctuation-dissipation relations II: Fluctuation-dissipation theorem
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     A. Set-up
            Onsager’s principle, Derivation
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            Fourier and quantum versions
     B. Derivation
        {(#) a followup handout, prob. dist'n in presence of field}
     C. Comments on FDT
         1. Kubo formula  2. Onsager relations
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UNIT IV. POLYMERS AND ENTROPIC FORCES
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Overview handout: "Soft condensed matter: where physics meets biology"
    by Poon, McLeish, and Donald (from Physics World, May 2001)

4.1 Overview
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        A. some familiar polymers as plastics (#)
        B. heteropolymers in bio (DNA, RNA, proteins) (#)
        C. embedding a manifold in a higher dimensional space (#)
           (#) figures (from Chaikin & Lubensky) of lipids (membrane formers)
        D. Depletion force (ex. of entropic force)
              1) d=1 intervals, integrate out every second one
              2) fluid with a large sphere and many small particles

4.2 Ideal polymers
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        A. : length N monomers, strain r(Na)/Na, forces
           d=1 toy model freely jointed chain - like folding yardstick
           entropy and effective spring constant
        B. Jointed chains (d=3_
                  (0) freely jointed chain;
                  (1) jointed chain w/interactions equiv. to Heisenberg spin chain
                  (2) persistence length
                  (3) realistic tetrahedrally bonded chain (trans/gauche) chain
            Persistence length
        C. Worm-like chain (continuum)
            elastic form, tangent vector t(s), pers. length = el. const/T
            arc length, eff. monomer length
           {Twist elasticity too}

4.3 Self-interacting polymers
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        A. Flory theory
            derivation as exclusion (or as repulsion energy u)
            "excluded volume parameter" v
        B. Theta point: With attractive term
            analogy to lattice gas models of phase sep.
            regimes of v: "athermal" "good solvent" "theta-point" "poor solvent"
        C. Critical exponents [of self-avoiding walk]
            exponents nu, gamma
            Force pulling SAW: not Hooke's law!
        D. Helix-coil transition in DNA <brief>
        E. Measuring the geometry of a polymer
             exponent nu, real-space dist. as fcn. of arc length
             {Fractal dim. is 1/nu}
            1) diffraction (structure factor)
            2) viscosity (or diffusion const.)
      @ images from Zallen text
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UNIT V. D=2 CONTINUUM
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5.1 2D XY model: correlations and vortices
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   A. XY model
         Examples:  Kosterlitz-Thouless  systems (table) #
                 further notes on locking (table) #
   B. Fluctuations and correlations
       1. Derive angle fluctuations (~log R)
       2.  Spin correlation function (power law, QLRO)
   C. Vortices
            1. energy of vortices
            2. Electrostatic analogy (#)


5.2. Kosterlitz-Thouless R.G.
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   A. Thermal vortex pairs
          1. separates vortex pair interction + Gaussian background
          2. Bound vortex pairs
          3. remormalized stiffness (dipoles => polarization)
   B. [Kosterlitz-Thouless] Renormalization group @
   C: Spin anisotropy in the XY model (d=2) @


5.3. Interfaces and Roughening  [MOSTLY OMITTED]
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    A. Basic ideas (#)
         surface free energy, height h(r), height correl. C(r) "Roughness")
    B. "Solid-on-solid" models (SOS) (#)
    C. Rough behavior (#)
          rationalization of  [gradient-squared in d=1+1]
    D. Roughening  transition (d=2+1)
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